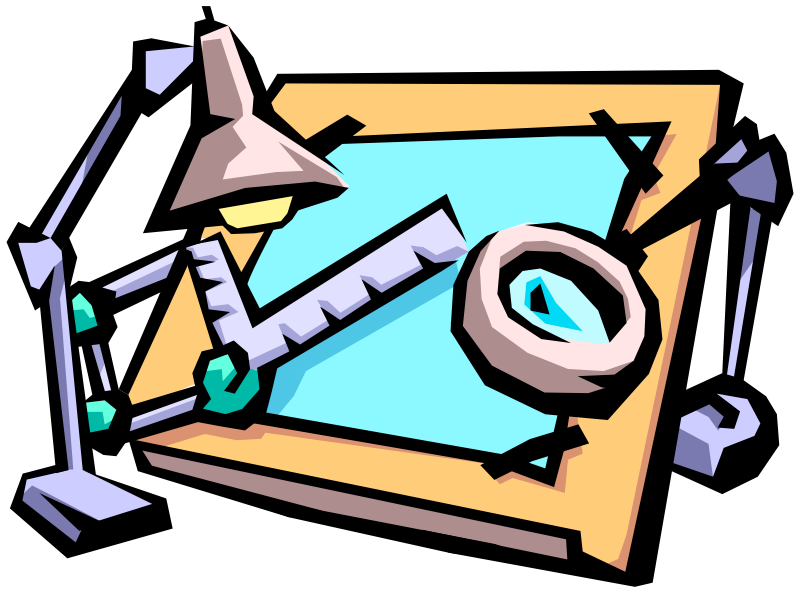
Sequential circuit design

- Yesterday we focused on analyzing sequential circuits.
  - We found the outputs and next states for all possible combinations of inputs and current states.
  - This information is summarized in a state table or a state diagram.
- Today we'll give some examples of sequential circuit design.
  - We first make a state table or diagram to express the computation.
  - Then we can turn that table or diagram into a sequential circuit.



Sequential circuit design procedure

1. Make a state table based on the problem statement. The table should show the present states, inputs, next states and outputs. (It may be easier to draw a state diagram first, and then convert that to a table.)

2. Assign binary codes to all of the possible states. If you have n states, the binary codes will need at least $\lceil \log_2 n \rceil$ digits, and the circuit will have at least $\lceil \log_2 n \rceil$ flip-flops.

3. For each flip-flop and each row of the state table, find the flip-flop input values that will generate the next state from the present state. You can use flip-flop excitation tables to help you here.

4. Derive simplified equations for the flip-flop inputs and the outputs.

5. Build the circuit!

Sequence recognizers

- A sequence recognizer looks for a special bit pattern in some input.
- The recognizer circuit has only one input, X.
  - One bit of input is supplied on each clock cycle. For example, it would take 20 clock cycles to enter and scan a 20-bit sequence.
  - This is an easy way to permit arbitrarily long input sequences.
- There is one output Z, which is 1 when the desired pattern is found.
- Our example will detect the bit pattern "1001".

Inputs:	1 1 1 0 0 1 1 0 1 0 0 1 0 0 1 1 0…
Outputs:	0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0 0…

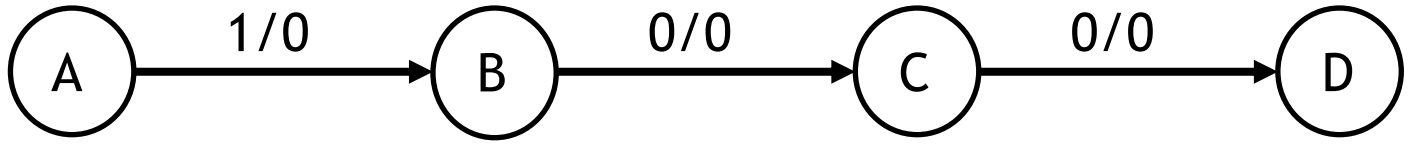
  Here, one input and one output bit appear on each clock cycle.
- This requires a sequential circuit because the circuit has to "remember" the inputs from previous clock cycles, in order to determine whether or not a match is found.

Step 1: Making a state table

- The first thing you have to figure out is precisely how the use of state will help you solve the given problem.
  - Make a state table based on the problem statement. The table should show the present states, inputs, next states and outputs.
  - Sometimes it may be easier to come up with a state diagram first and then convert that to a table.
- This is usually the most difficult step. Once you have the state table, the rest of the design procedure is the same for all sequential circuits.
- Sequence recognizers are one of the harder examples we will see in this class, so if you understand this you're in good shape.

A basic state diagram

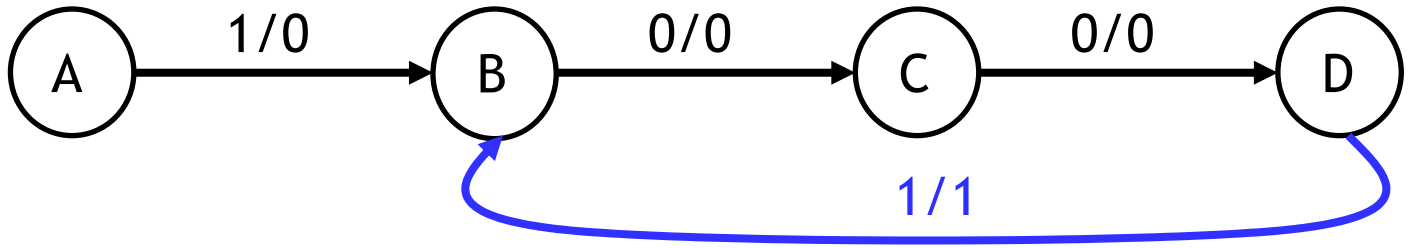
- What state do we need for the sequence recognizer?
  - We have to "remember" inputs from previous clock cycles.
  - For example, if the previous three inputs were 100 and the current input is 1, then the output should be 1.
  - In general, we will have to remember occurrences of parts of the desired pattern—in this case, 1, 10, and 100.
- We'll start with a basic state diagram.



State	Meaning
A	None of the desired pattern (1001) has been entered yet
B	We've already seen the first bit (1) of the desired pattern
C	We've already seen the first two bits (10) of the desired pattern
D	We've already seen the first three bits (100) of the desired pattern

Overlapping occurrences of the pattern

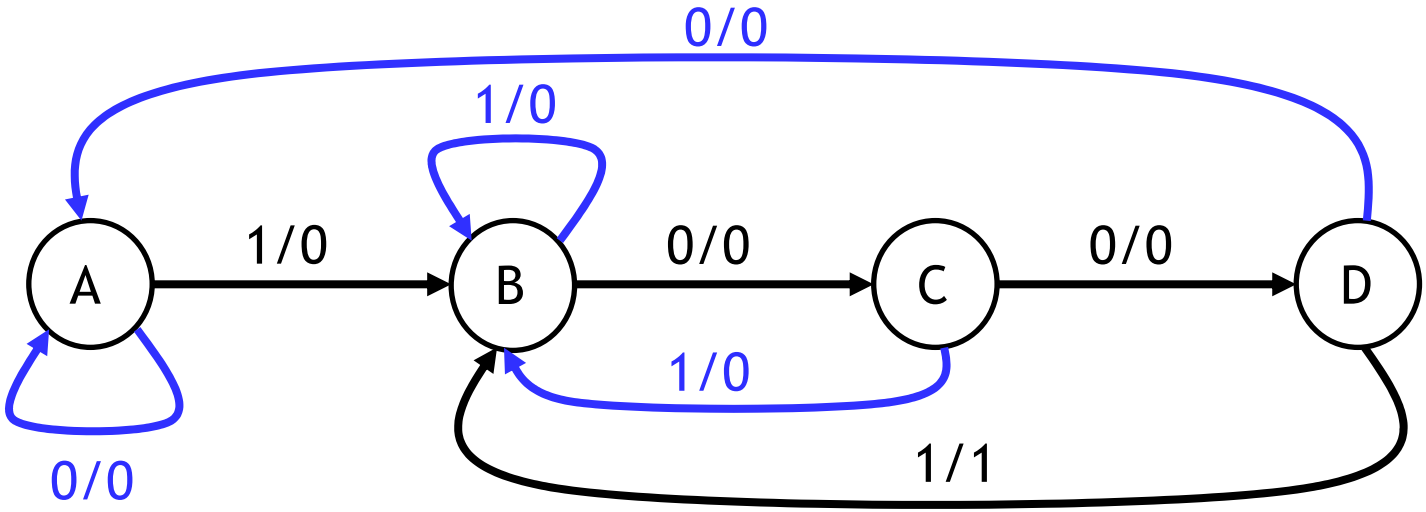
- What happens if we're in state D (the last three inputs were 100), and the current input is 1?
  - The output should be a 1, because we've found the desired pattern.
  - But this last 1 could also be the start of another occurrence of the pattern! For example, 1001001 contains two occurrences of 1001.
  - To properly detect overlapping occurrences of the pattern, the next state should be B.



State	Meaning
A	None of the desired pattern (1001) has been entered yet
B	We've already seen the first bit (1) of the desired pattern
C	We've already seen the first two bits (10) of the desired pattern
D	We've already seen the first three bits (100) of the desired pattern

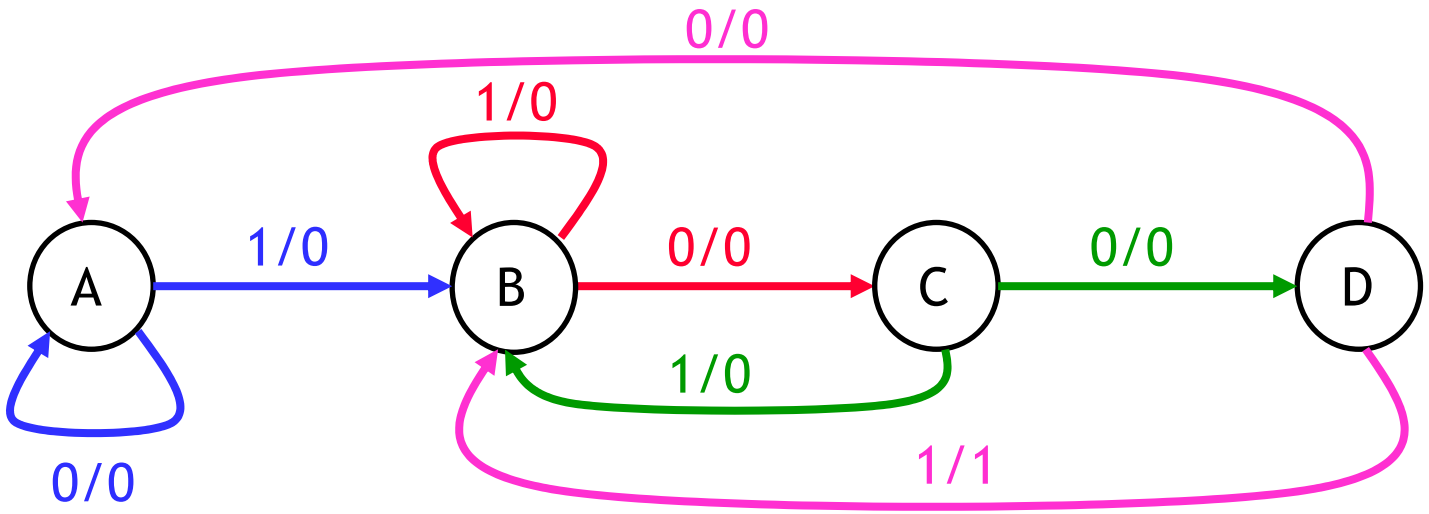
Filling in the other arrows

- Remember that we need *two* outgoing arrows for each node, to account for the two input possibilities of X = 0 and X = 1.
- The remaining arrows we need are shown in blue. They also allow for the correct detection of overlapping occurrences of 1001.

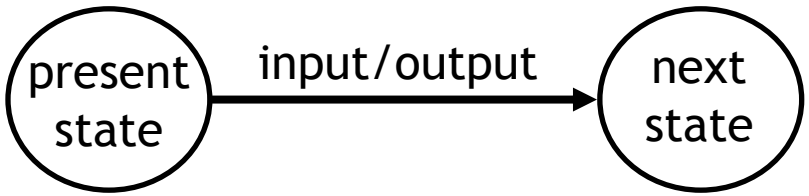


State	Meaning
A	None of the desired pattern (1001) has been entered yet
B	We've already seen the first bit (1) of the desired pattern
C	We've already seen the first two bits (10) of the desired pattern
D	We've already seen the first three bits (100) of the desired pattern

Finally, making the state table



Remember how the state diagram arrows correspond to rows of the state table.

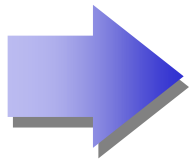


Present State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	B	1

Step 2: Assigning binary codes to states

- We have four states ABCD, so we need at least two flip-flops Q_1Q_0.
- The easiest thing is to represent state A with Q_1Q_0 = 00, B with 01, C with 10, and D with 11. (You could have used these codes in Step 1 too, rather than using temporary state labels like ABCD.)
- The state assignment can have a big impact on circuit complexity, but we won't worry about that too much in this class.

Present State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	B	1



Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Output Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	0	1	1

Step 3: Finding flip-flop input values

- Next we have to figure out how to actually make the flip-flops change from their present state into the desired next state.
- This depends on what kind of flip-flops you use! We'll use two JKs here.
- For each flip-flip Q_i, look at its present and next states, and determine what the inputs J_i and K_i should be in order to make that state change.

Present State		Input	Next State		Flip-flop Inputs				Output
Q_1	Q_0	X	Q_1	Q_0	J_1	K_1	J_0	K_0	Z
0	0	0	0	0					0
0	0	1	0	1					0
0	1	0	1	0					0
0	1	1	0	1					0
1	0	0	1	1					0
1	0	1	0	1					0
1	1	0	0	0					0
1	1	1	0	1					1

Finding JK flip-flop input values

- For JK flip-flops, this is a little tricky. Recall the JK characteristic table.

J	K	Q(t+1)	Operation
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

- If the present state of a JK flip-flop is 0 and we want the next state to be 1, then we have *two* choices for the JK inputs.
  - We can use JK = 10, to explicitly set the flip-flop's next state to 1.
  - We can also use JK = 11, to complement the current state 0.
- So to change from 0 to 1 we must set J = 1, but K could be *either* 0 or 1.
- Similarly, the other three possible state transitions can all be done in two different ways as well.

JK excitation table

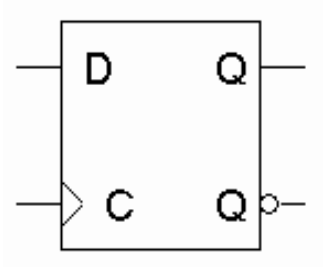
- An excitation table shows what flip-flop inputs are required in order to make a desired state change.

Q(t)	Q(t+1)	J	K	Operation
0	0	0	x	No change/Reset
0	1	1	x	Set/Complement
1	0	x	1	Reset/Complement
1	1	x	0	No change/Set

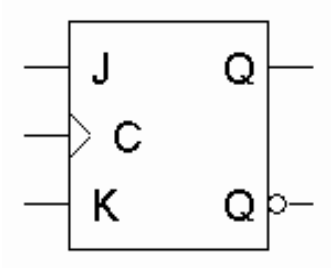
- This is the same information that's given in the characteristic table, but presented "backwards."

J	K	Q(t+1)	Operation
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

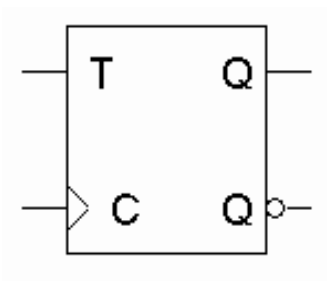
Excitation tables for all flip-flops



Q(t)	Q(t+1)	D	Operation
0	0	0	Reset
0	1	1	Set
1	0	0	Reset
1	1	1	Set



Q(t)	Q(t+1)	J	K	Operation
0	0	0	x	No change/Reset
0	1	1	x	Set/Complement
1	0	x	1	Reset/Complement
1	1	x	0	No change/Set



Q(t)	Q(t+1)	T	Operation
0	0	0	No change
0	1	1	Complement
1	0	1	Complement
1	1	0	No change

Back to the example

- We can now use the JK excitation table on the right to find the correct values for each flip-flop's inputs, based on its present and next states.

Q(t)	Q(t+1)	J	K
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Flip-flop Inputs J_1	Flip-flop Inputs K_1	Flip-flop Inputs J_0	Flip-flop Inputs K_0	Output Z
0	0	0	0	0	0	x	0	x	0
0	0	1	0	1	0	x	1	x	0
0	1	0	1	0	1	x	x	1	0
0	1	1	0	1	0	x	x	0	0
1	0	0	1	1	x	0	1	x	0
1	0	1	0	1	x	1	1	x	0
1	1	0	0	0	x	1	x	1	0
1	1	1	0	1	x	1	x	0	1

Step 4: Find equations for the FF inputs and output

- Now you can make K-maps and find equations for each of the four flip-flop inputs, as well as for the output Z.
- These equations are in terms of the present state and the inputs.
- The advantage of using JK flip-flops is that there are many don't care conditions, which can result in simpler MSP equations.

Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Flip-flop Inputs J_1	Flip-flop Inputs K_1	Flip-flop Inputs J_0	Flip-flop Inputs K_0	Output Z
0	0	0	0	0	0	x	0	x	0
0	0	1	0	1	0	x	1	x	0
0	1	0	1	0	1	x	x	1	0
0	1	1	0	1	0	x	x	0	0
1	0	0	1	1	x	0	1	x	0
1	0	1	0	1	x	1	1	x	0
1	1	0	0	0	x	1	x	1	0
1	1	1	0	1	x	1	x	0	1

$J_1 = X'Q_0$

$K_1 = X + Q_0$

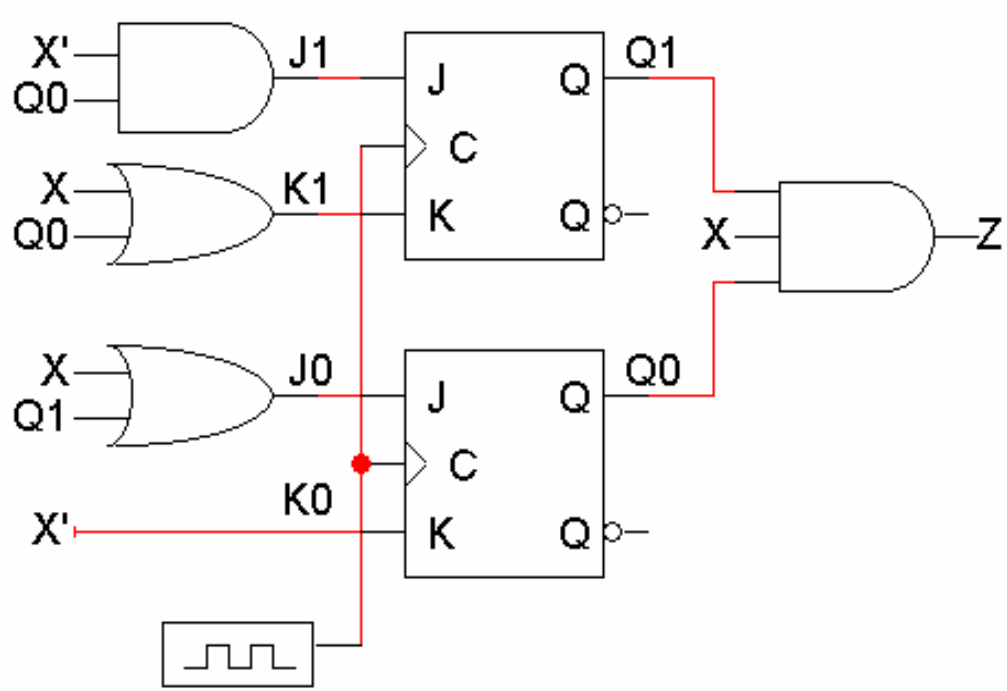
$J_0 = X + Q_1$

$K_0 = X'$

$Z = Q_1Q_0X$

Step 5: Build the circuit

- Hey! It's the same circuit we saw yesterday. Tricky Howard!
- Now you can see that this circuit detects occurrences of the pattern 1001 in a serial input stream X.



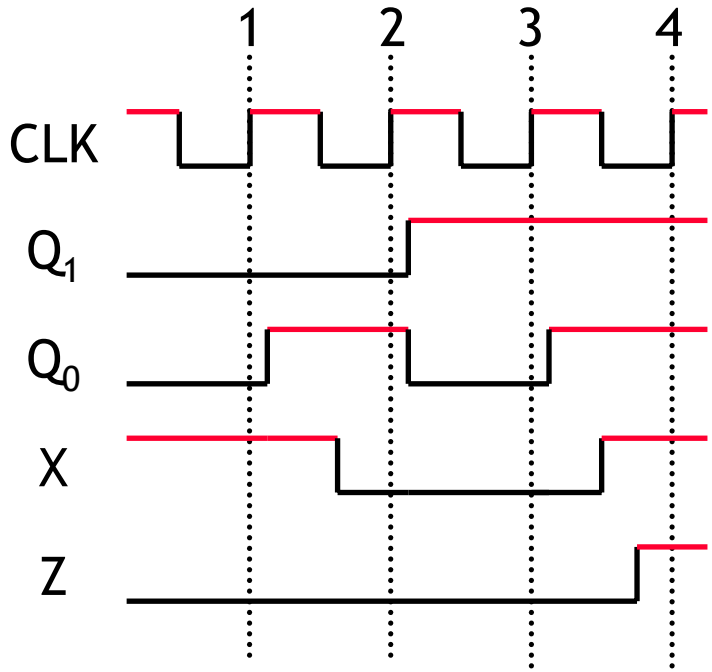
$J_1 = X'Q_0$
$K_1 = X + Q_0$

$J_0 = X + Q_1$
$K_0 = X'$

$Z = Q_1Q_0X$

Timing diagram

- Here is one example timing diagram for our sequence detector.
  - The flip-flops Q_1Q_0 start in the initial state, 00.
  - On the first three positive clock edges, X is 1, 0, and 0. These inputs cause Q_1Q_0 to change, so after the third edge Q_1Q_0 = 11.
  - Then when X = 1, Z becomes 1 also, meaning that 1001 was found.
- The output Z does *not* necessarily change at only positive clock edges; it can change whenever X changes, since $Z = Q_1Q_0X$.



Building the same circuit with D flip-flops

- What if you want to build the circuit using D flip-flops instead?
- We already have the state table and state assignments, so we can just start from Step 3, finding the flip-flop input values.
- D flip-flops have only one input, so our table only needs two columns for D_1 and D_0.

Present State Q_1	Q_0	Input X	Next State Q_1	Q_0	Flip-flop Inputs D_1	D_0	Output Z
0	0	0	0	0			0
0	0	1	0	1			0
0	1	0	1	0			0
0	1	1	0	1			0
1	0	0	1	1			0
1	0	1	0	1			0
1	1	0	0	0			0
1	1	1	0	1			1

D flip-flop input values (Step 3)

- The D excitation table is pretty boring; you just set the D input to whatever the next state should be.
- You don't even need to show separate columns for D_1 and D_0—they will always be the same as the Next State columns.

Q(t)	Q(t+1)	D	Operation
0	0	0	Reset
0	1	1	Set
1	0	0	Reset
1	1	1	Set

Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Flip-flop Inputs D_1	Flip-flop Inputs D_0	Output Z
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	1	0	1	0	0
0	1	1	0	1	0	1	0
1	0	0	1	1	1	1	0
1	0	1	0	1	0	1	0
1	1	0	0	0	0	0	0
1	1	1	0	1	0	1	1

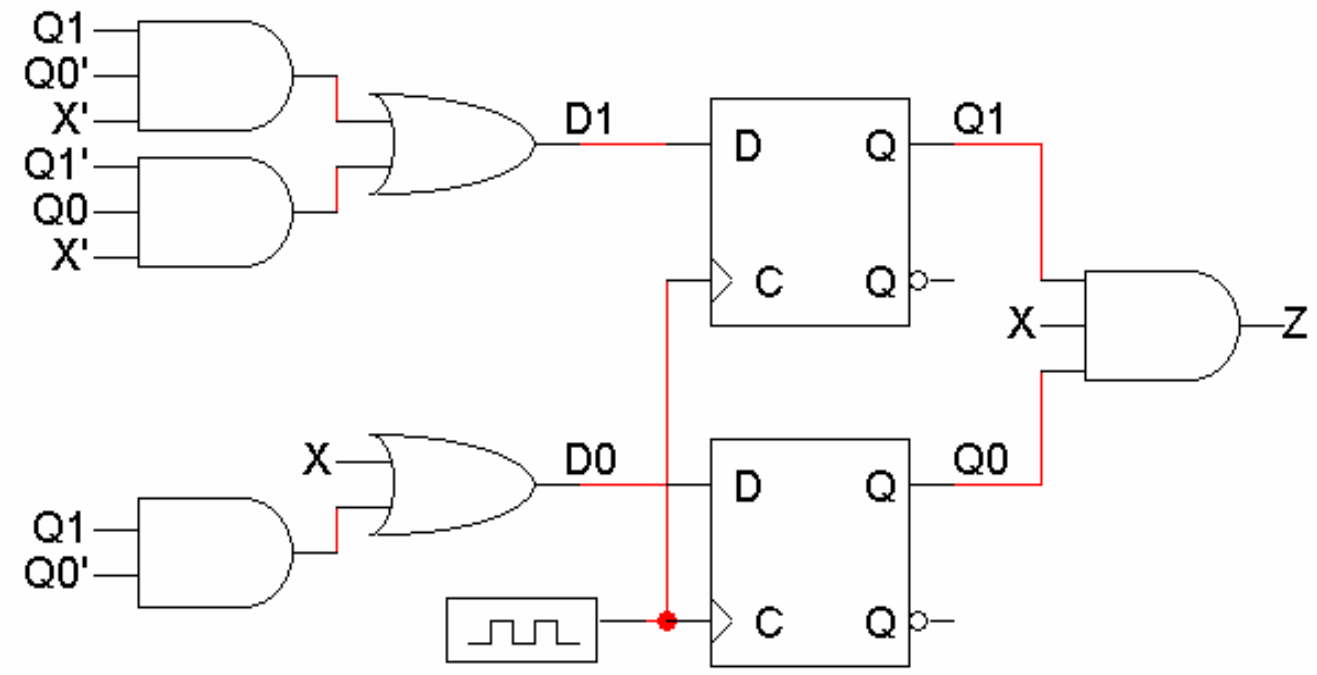
Finding equations (Step 4)

- If you use K-maps again, you should find the following equations.

$$D_1 = Q_1Q_0'X' + Q_1'Q_0X'$$
$$D_0 = X + Q_1Q_0'$$
$$Z = Q_1Q_0X$$

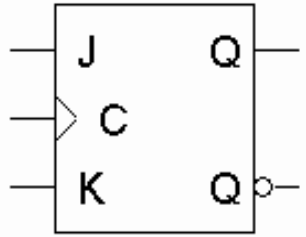
Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Flip-flop Inputs D_1	Flip-flop Inputs D_0	Output Z
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	1	0	1	0	0
0	1	1	0	1	0	1	0
1	0	0	1	1	1	1	0
1	0	1	0	1	0	1	0
1	1	0	0	0	0	0	0
1	1	1	0	1	0	1	1

Building the circuit (Step 5)

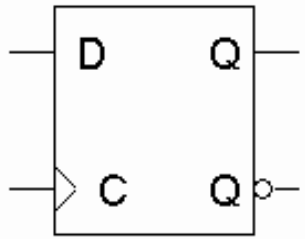


Flip-flop comparison

- JK flip-flops are good because there are many don't care values in the flip-flop inputs, which can lead to a simpler circuit.



- D flip-flops have the advantage that you don't have to set up flip-flop inputs at all, since Q(t+1) = D. However, the D input equations are usually more complex than JK input equations.



- In practice, D flip-flops are used more often.
  - There is only one input for each flip-flop, not two.
  - There are no excitation tables to worry about.
  - D flip-flops themselves are simpler to implement than JK flip-flops.

Summary

- The basic sequential circuit design procedure has five steps.
  1. Make a state table and, if needed, a state diagram.
  2. Assign binary codes to the states (if you didn't already).
  3. Use the present states, next states, and flip-flop excitation tables to find the correct flip-flop input values.
  4. Write simplified equations for the flip-flop inputs and outputs.
  5. Build the circuit.
- Tomorrow we'll look at more examples of sequential circuits, including different types of registers and counters.